

17-2

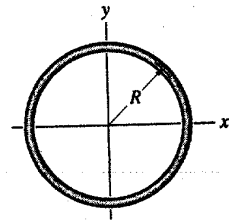
Determine the moment of inertia of the thin ring about the  $z$  axis. The ring has a mass  $m$ .

$$I_z = \int_0^{2\pi} \rho A (R d\theta) R^2 = 2\pi \rho A R^3$$

$$m = \int_0^{2\pi} \rho A R d\theta = 2\pi \rho A R$$

Thus,

$$I_z = m R^2 \quad \text{Ans}$$



17-12

Determine the moment of inertia about an axis perpendicular to the page and passing through the pin at  $O$ . The thin plate has a hole in its center. Its thickness is 50 mm, and the material has a density  $\rho = 50 \text{ kg/m}^3$ .

$m_p = 50(1.40)^2(0.050) = 4.90 \text{ kg}$   
 $m_h = (50)(\pi)(0.15)^2(0.050) = 0.1767 \text{ kg}$   
 $I_O = \frac{1}{12}(4.90)[(1.40)^2 + (1.40)^2] + 4.90(0.990)^2 - [\frac{1}{2}(0.1767)(0.15)^2 + (0.1767)(0.990)^2]$   
 $I_O = 6.23 \text{ kg} \cdot \text{m}^2 \quad \text{Ans}$

17-16

The pendulum consists of the 3-kg slender rod and the 5-kg thin plate. Determine the location  $\bar{y}$  of the center of mass  $G$  of the pendulum; then calculate the moment of inertia of the pendulum about an axis perpendicular to the page and passing through  $G$ .

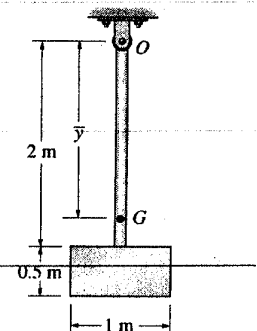
$$\bar{y} = \frac{\sum \bar{y}m}{\sum m} = \frac{1(3) + 2.25(5)}{3+5} = 1.781 \text{ m} = 1.78 \text{ m} \quad \text{Ans}$$

$$I_G = \sum \bar{I}_G + md^2$$

$$= \frac{1}{12}(3)(2)^2 + 3(1.781 - 1)^2 + \frac{1}{12}(5)(0.5^2 + 1^2) + 5(2.25 - 1.781)^2$$

$$= 4.45 \text{ kg} \cdot \text{m}^2$$

Ans



The fork lift has a boom with mass  $M_1$  and a mass center at  $G$ . If the vertical acceleration of the boom is  $a_G$ , determine the horizontal and vertical reactions at the pin  $A$  and on the short link  $BC$  when the load  $M_2$  is lifted.

Units Used:

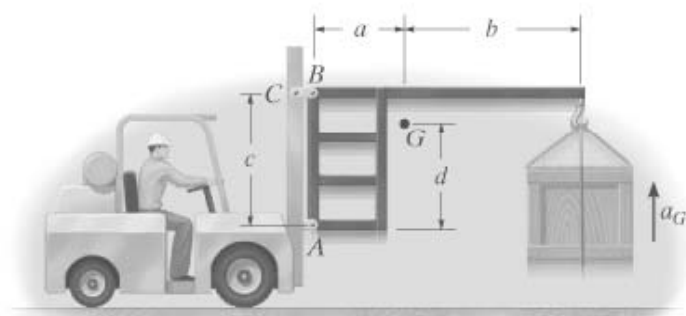
$$Mg = 10^3 \text{ kg} \quad \text{kN} = 10^3 \text{ N}$$

Given:

$$M_1 = 800 \text{ kg} \quad a = 1 \text{ m}$$

$$M_2 = 1.25 \text{ Mg} \quad b = 2 \text{ m} \\ c = 1.5 \text{ m}$$

$$a_G = 4 \frac{\text{m}}{\text{s}^2} \quad d = 1.25 \text{ m}$$



Solution:

Guesses

$$A_x = 1 \text{ N} \quad F_{CB} = 1 \text{ N}$$

$$A_y = 1 \text{ N}$$

Given

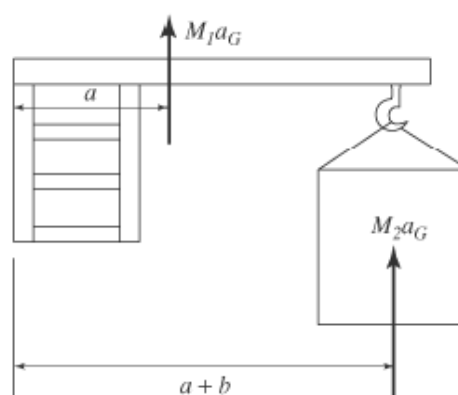
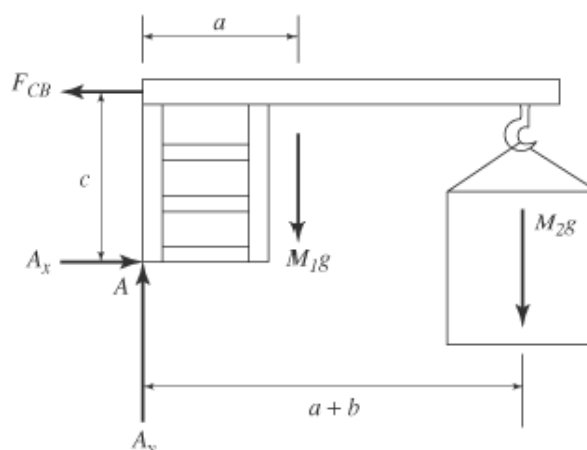
$$-F_{CB} + A_x = 0$$

$$A_y - (M_1 + M_2)g = (M_1 + M_2)a_G$$

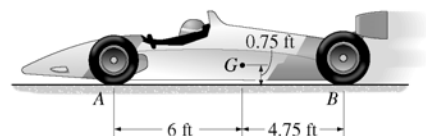
$$F_{CB}c - M_1ga - M_2g(a+b) = M_1a_Ga + M_2a_G(a+b)$$

$$\begin{pmatrix} A_x \\ A_y \\ F_{CB} \end{pmatrix} = \text{Find}(A_x, A_y, F_{CB})$$

$$\begin{pmatrix} A_x \\ A_y \\ F_{CB} \end{pmatrix} = \begin{pmatrix} 41.9 \\ 28.3 \\ 41.9 \end{pmatrix} \text{ kN}$$



Determine the maximum acceleration that can be achieved by the car without having the front wheels *A* leave the track or the rear drive wheels *B* slip on the track. The coefficient of static friction is  $\mu_s = 0.9$ . The car's mass center is at *G*, and the front wheels are free to roll. Neglect the mass of all the wheels.



**Equations of Motion:**

$$\leftarrow \Sigma F_x = m(a_G)_x; \quad F_B = \frac{1550}{32.2}a \quad (1)$$

$$+\uparrow \Sigma F_y = m(a_G)_y; \quad N_A + N_B - 1550 = 0 \quad (2)$$

$$\curvearrowleft \Sigma M_G = 0; \quad N_B(4.75) - F_B(0.75) - N_A(6) = 0 \quad (3)$$

If we assume that the front wheels are about to leave the track,  $N_A = 0$ . Substituting this value into Eqs. (2) and (3) and solving Eqs. (1), (2), (3),

$$N_B = 1550 \text{ lb} \quad F_B = 9816.67 \text{ lb} \quad a = 203.93 \text{ ft/s}^2$$

Since  $F_B > (F_B)_{\max} = \mu_s N_B = 0.9(1550) \text{ lb} = 1395 \text{ lb}$ , the rear wheels will slip. Thus, the solution must be reworked so that the rear wheels are about to slip.

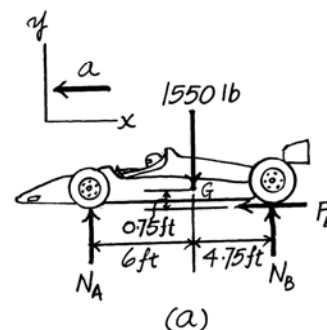
$$F_B = \mu_s N_B = 0.9N_B \quad (4)$$

Solving Eqs. (1), (2), (3), and (4) yields

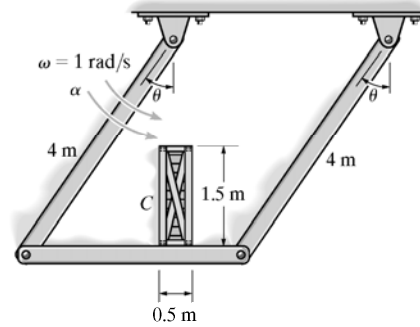
$$N_A = 626.92 \text{ lb} \quad N_B = 923.08 \text{ lb}$$

$$a = 17.26 \text{ ft/s}^2 = 17.3 \text{ ft/s}^2$$

**Ans.**



The 50-kg uniform crate rests on the platform for which the coefficient of static friction is  $\mu_s = 0.5$ . If at the instant  $\theta = 30^\circ$  the supporting links have an angular velocity  $\omega = 1 \text{ rad/s}$  and angular acceleration  $\alpha = 0.5 \text{ rad/s}^2$ , determine the frictional force on the crate.



**Curvilinear Translation:**

$$(a_G)_n = (1)^2(4) = 4 \text{ m/s}^2$$

$$(a_G)_t = 0.5(4) \text{ m/s}^2 = 2 \text{ m/s}^2$$

$$\rightarrow \Sigma F_x = m(a_G)_x; \quad F_C = 50(4) \sin 30^\circ + 50(2) \cos 30^\circ$$

$$+\uparrow \Sigma F_y = m(a_G)_y; \quad N_C - 50(9.81) = 50(4) \cos 30^\circ - 50(2) \sin 30^\circ$$

Solving,

$$F_C = 186.6 \text{ N}$$

$$N_C = 613.7 \text{ N}$$

$$(F_C)_{\max} = 0.5(613.7) = 306.9 \text{ N} > 186.6 \text{ N}$$

**OK**

$$\zeta + \Sigma M_G = \Sigma (M_k)_G; \quad N_C(x) - F_C(0.75) = 0$$

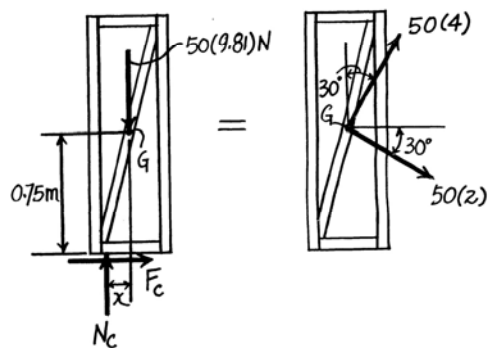
$$613.7(x) - 186.6(0.75) = 0$$

$$x = 0.228 \text{ m} < 0.25 \text{ m}$$

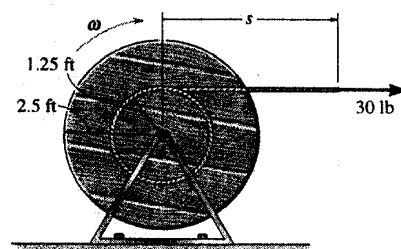
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$$\text{Thus, } F_C = 187 \text{ N}$$

**Ans.**



A cord is wrapped around the inner core of a spool. If the cord is pulled with a constant tension of 30 lb and the spool is originally at rest, determine the spool's angular velocity when  $s = 8$  ft of cord has unwound. Neglect the weight of the 8-ft portion of cord. The spool and the entire cord have a total weight of 400 lb, and the radius of gyration about the axle  $A$  is  $k_A = 1.30$  ft.



$$I_A = mk_A^2 = \left(\frac{400}{32.2}\right)(1.30)^2 = 20.99 \text{ slug} \cdot \text{ft}^2$$

$$\sum M_A = I_A \alpha: \quad 30(1.25) = 20.99(\alpha) \quad \alpha = 1.786 \text{ rad/s}^2$$

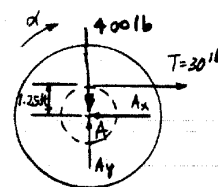
$$\text{The angular displacement is } \theta = \frac{s}{r} = \frac{8}{1.25} = 6.4 \text{ rad.}$$

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

$$\omega^2 = 0 + 2(1.786)(6.4 - 0)$$

$$\omega = 4.78 \text{ rad/s}$$

Ans



A cord having negligible mass is wrapped over the 15-lb disk at *A* and passes over the 5-lb disk at *B*. If a 3-lb block *C* is attached to its end and released from rest, determine the speed of the block after it descends 3 ft. Also, what is the tension in the horizontal and vertical segments of the cord? Assume no slipping of the cord over the disk at *B*. Neglect friction at the pins *D* and *E*.

Disk *A* :

$$(\sum M_D = I_D \alpha_A; \quad T_A (1.5) = \left[ \frac{1}{2} \left( \frac{15}{32.2} \right) (1.5)^2 \right] \alpha_A \quad (1)$$

Disk *B* :

$$(\sum M_E = I_E \alpha_B; \quad T_C (0.5) - T_A (0.5) = \left[ \frac{1}{2} \left( \frac{5}{32.2} \right) (0.5)^2 \right] \alpha_B$$

Block *C* :

$$+\downarrow \sum F_y = m(a_G)_y; \quad 3 - T_C = \left( \frac{3}{32.2} \right) a_C \quad (3)$$

$$\alpha_B = \frac{a_C}{0.5} \quad (4)$$

$$\alpha_A = \frac{a_C}{1.5} \quad (5)$$

Substituting Eqs. 1, 3, 4 and 5 into Eq. 2,

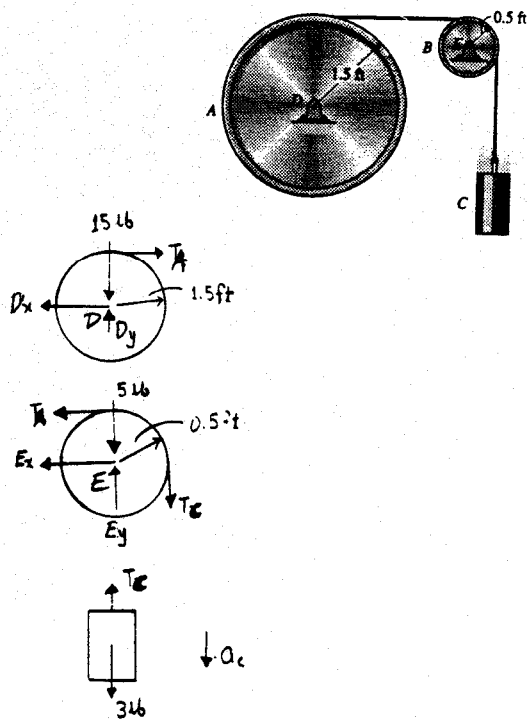
$$\left( 3 - \frac{3}{32.2} a_C \right) (0.5) - \frac{1}{2} \left( \frac{15}{32.2} \right) (1.5) \left( \frac{a_C}{1.5} \right) (0.5) = \left[ \frac{1}{2} \left( \frac{5}{32.2} \right) (0.5)^2 \right] \left( \frac{a_C}{0.5} \right)$$

$$a_C = 7.43 \text{ ft/s}^2$$

Thus

$$T_A = 1.73 \text{ lb} \quad \text{Ans}$$

$$T_C = 2.31 \text{ lb} \quad \text{Ans}$$



$$(+\downarrow) \quad v^2 = v_0^2 + 2a_C(s - s_0)$$

$$v_C^2 = 0 + 2(7.43)(3 - 0)$$

$$v_C = 6.68 \text{ ft/s} \quad \text{Ans}$$